Name: _____

SM3 13.2: Geometric Sequences

Memorize: General Rule: $a_n = a_1 \cdot r^{n-1}$ Recursive Rule: $a_n = r \cdot a_{n-1}$

Vocab: a: a sequence of numbers n: the number of terms in a sequence a

a_1: the first term of a sequence a a_n: the n^{th} term of sequence a

r: the common ratio of a sequence a; multiply by r to get from any term to the next term; divide by r to get from any term to the previous term.

Let $a = 2, 8, 32, \ldots$. Evaluate the following:

1) $a_1 = 2$ 2) $a_3 = 32$ 3) $a_5 = 512$

4) $a_8 = 32768$ 5) $n = \infty$ 6) $r = 4$

Let $b = \frac{1}{3}, -\frac{1}{9}, \frac{1}{27}, -\frac{1}{81}, \ldots$. Evaluate the following:

7) $b_1 = \frac{1}{3}$ 8) $b_3 = \frac{1}{27}$ 9) $b_5 = \frac{1}{243}$

10) $b_{12} = -\frac{1}{531441}$ 11) $n = \infty$ 12) $r = -\frac{1}{3}$

Let $c = \frac{5}{8}, \frac{5}{4}, \ldots, 80$. Evaluate the following:

13) $c_1 = \frac{5}{8}$ 14) $c_3 = \frac{5}{2}$ 15) $c_5 = 10$

16) $c_7 = 40$ 17) $n = 8$ 18) $r = 2$

For problems 19-28, write the explicit formula for the sequence $a_n = a_1 \cdot r^{n-1}$ and find the indicated term.

19) $h = 2,8,32,128, \ldots$

$h_n = 2(4)^{n-1}$

$h_{10} = 524288$

20) $k = 0.5,1,2,4, \ldots$

$k_n = 0.5(2)^{n-1}$

$k_{11} = 512$

21) $m = -\frac{27}{4}, \frac{9}{2}, -3,2, \ldots$

$m_n = -\frac{27}{4}\left(-\frac{2}{3}\right)^{n-1}$

$m_{10} = \frac{128}{729}$

22) $p = -0.5, -2, -8, -32, \ldots$

$p_n = -0.5(4)^{n-1}$

$p_9 = -32768$

23) $q = 4,2,1, \frac{1}{2}, \ldots$

$q_n = 4\left(\frac{1}{2}\right)^{n-1}$

$q_{10} = \frac{1}{128}$

24) $s = -1,5, -25,125, \ldots$

$s_n = -(-5)^{n-1}$

$s_{10} = -390625$

25) $t = 1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots$

$t_n = \left(\frac{1}{3}\right)^{n-1}$

$t_{11} = \frac{1}{59049}$

26) $u = 2,10,50,250, \ldots$

$u_n = 2(5)^{n-1}$

$u_9 = 781250$

27) $v = -\frac{3}{2}, 1, -\frac{2}{3}, \frac{4}{9}, \ldots$

$v_n = -\frac{3}{2}\left(-\frac{2}{3}\right)^{n-1}$

$v_{12} = \frac{1024}{59049}$

28) $w = 0.25, -1,4, -16, \ldots$

$w_n = 0.25(-4)^{n-1}$

$w_{11} = 262144$

William deposits $5 into a savings account that has a 2% annual compound interest rate. Let c represent the amount of money in the account during the n^{th} year that the account is open.

29) Is c a geometric sequence? Justify your response.

Yes; there is a common ratio between terms.

30) $c_1 = 5$ 31) $r = 1.02$

32) How much money does William have after 5 years have passed? $\$5.52$

33) How much money does William have after 25 years have passed? $\$8.20$

A group of 500 students begin their mathematical career that will consist of SM1, SM2, SM3, Calc, and Stats. Each year, 10% of the students fail to complete the course. Let sequence w represent the number of students still in contention for having never failed a course where w_n represents number of students in the n^{th} class of the path listed above. If a fraction of a student fails the course, round up and have the whole student fail the course (we can't very well fail your arms but pass your legs, can we?).

34) Is w a geometric sequence? Justify your response.

Yes; there is a common ratio between terms.

35) $w_1 = 500$ 36) $r = 0.9$

37) Write sequence w: $500,\ 450,\ 405,\ 364,\ 328$

38) What percent of the initial group of students pass Stats? $\frac{295}{500} = 59\%$

39) If there are 5 courses that could be failed, and we lose 10% of the 500 students each year, why are there not 250 students passing Stats?

10% of the previous year's students fail; this is not the same as 10% of the starting student population.

To help his students learn to persevere in tough testing environments, Mr. Wytiaz turns off the heat to the classroom during a unit test. The room begins at $70°\text{F}$, but the temperature falls by 3% every minute! Let t represent the temperature in the room after n minutes have passed, rounded to the nearest tenth.

40) $t_1 = 67.9$ 41) $t_2 = 65.9$

42) $t_5 = 60.1$ 43) $r = 0.97$

44) When will the room reach a temperature of $50°\text{F}$ or colder? Shortly after the 11^{th} minute.